

S.E Sem IV (ECS) 'C' ^{Summer} Scheme 2025 Examination

13/5/25

(3 Hours)

[Total Marks: 80]

N.B.: 1) Question No. 1 is Compulsory.

2) Answer any **THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Find (i) k (ii) mean and variance of the following distribution (5)

X	8	12	16	20	24
P(X)	1/12	5/12	k	1/4	1/12

(b) Evaluate $\int_c (x - iy^2) dz$ where c is curve $x = y^2$ from point (0,0) to (1,1). (5)(c) Find the extremal of $\int_0^{3\pi/2} (y^2 - y'^2) dx$ where $y(0)=0$; $y(3\pi/2)=1$ (5)

(d) State and verify the Cauchy Schwarz for the vectors (2, 1, 1,-1) and (1,-2, 1, 1). (5)

Q.2 (a) Evaluate $\int_c \frac{z}{(z^2+3z+2)} dz$, where c is the circle $|z + 1| = 1/2$. (6)

(b) Fit the second degree polynomial for following data (6)

Price(X)	20	16	10	11	16
Demand(Y)	22	41	120	89	56

(c) Transform the basis $\{(1,0,1) ; (1,1,0);(0,1,1)\}$ into orthogonal basis using Gram-Schmidt process. (8)Q.3 (a) Check whether the following sets are subspace of \mathbb{R}^3 (6)(i) $W = \{a, 0, 0\} / a \text{ belongs to } \mathbb{R}$ (ii) $W = \{(x, y, z) / x^2 + y^2 + z^2 \leq 1\}$

(b) Calculate the rank correlation coefficient 'R' for the following data (6)

x	10	12	18	16	15	40
y	12	18	20	15	50	25

(c) Obtain all possible Laurent's series expansion of $f(z) = \frac{1}{z^2+12z+35}$ about $z = 0$. (8)

- Q.4 (a) If X and Y are independent random variable with $E(X) = 6$ and $E(Y) = -6$, $V(X) = 4$, $V(Y) = 9$ then find (6)
- $E(2X + 3Y - 2)$
 - $V(3X + 2Y + 2)$

(b) Evaluate $\int_0^{1+i} (x - y + ix^2) dz$ along the line from $z = 0$ to $z = 1 + i$. (6)

- (c) Find rank, index, signature and nature of the Quadratic form by reducing it into Canonical form by congruent transformation $x^2 + 3y^2 + 3z^2 - 2yz$. (8)

- Q.5 (a) Three factories A, B, C produce 30%, 50% and 20% of the total production of an item. Out of their production 80%, 50% and 10% are defective respectively. An item is chosen at random and found to be defective. Find the probability that it was produced by the factory A. (6)

- (b) A continuous random variable has pdf $f(x) = k(x - x^2)$, $0 \leq x \leq 1$. Determine k , mean, and variance. (6)

- (c) Using Rayleigh -Ritz method solve boundary value problem $\int_0^1 (2x^2 y - 4y^2 + y'^2) dx$, $y(0)=0$, $y(1)=0$. (8)

Q.6 (a)

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

Calculate the Karl Pearson's coefficient of correlation. (6)

- (b) Find the extremals of $\int_a^b (16y^2 - y''^2 + x^2) dx$. (6)

- (c) Given: $6y = 5x + 90$, $15x = 8y + 130$ are regression lines and $\sigma_x^2 = 16$ then find (i) mean of X and Y (ii) correlation coefficient (r) (iii) σ_y^2 . (8)